

Event-by-event Charged-particle Fluctuations
in
Central Pb-Pb Collisions at $\sqrt{s_{NN}} = 17 \text{ GeV}$

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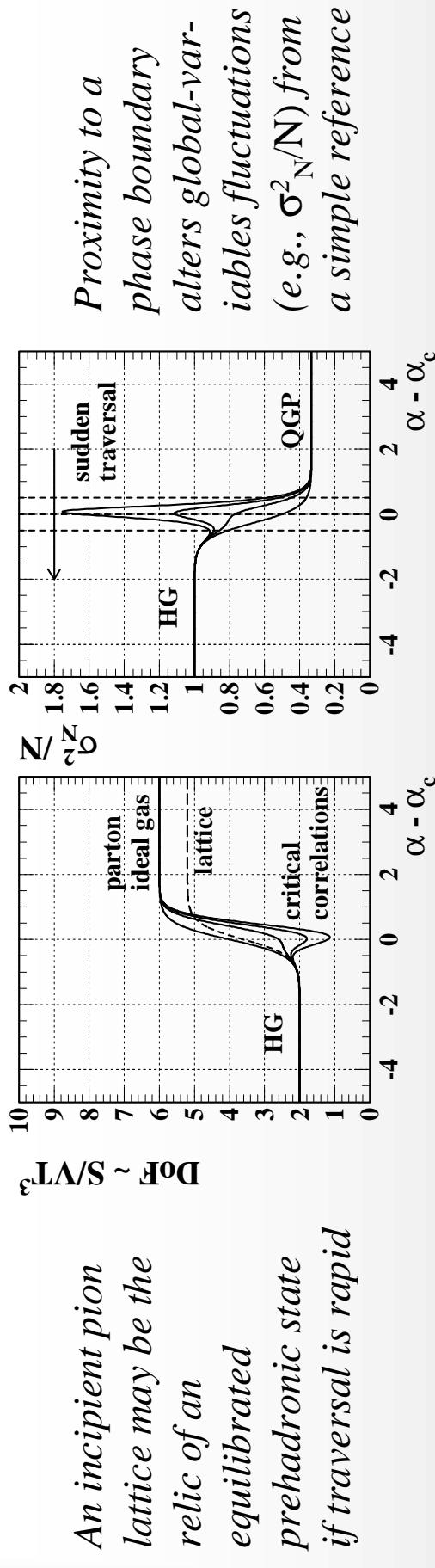
Quark Matter 2001

(NA49 Collaboration)

EbyE Global-variables Fluctuations

- Fluctuation/correlation studies
 - conducted in *transverse* phase space
- Search for QCD phase-boundary aspects in fluctuation systematics
 - Event ensemble treated as grand-canonical ensemble and subjected to *gaussian fluctuation analysis*
- Fluctuations represented by *full covariance matrix*
 - Resonance correlations
 - volume fluctuations
- Model-independent reference system based on the *central limit theorem*
 - Charge-dependent $N - \langle p_t \rangle$ correlations
 - Test independent particle production
- Global-variables sets: $(N, P_t, \langle p_t \rangle)_{+-c}$
 - Minimal sets of independent elements are $(N, \langle p_t \rangle)_{+-c}$
- Charge-dependent $\langle p_t \rangle$ fluctuations
 - Generalizes $\langle p_t \rangle$ analysis for cc case
 - Dynamical and isospin effects
- N_+, N_- fluctuations
 - volume fluctuations

Sudden Traversal of a Phase Boundary



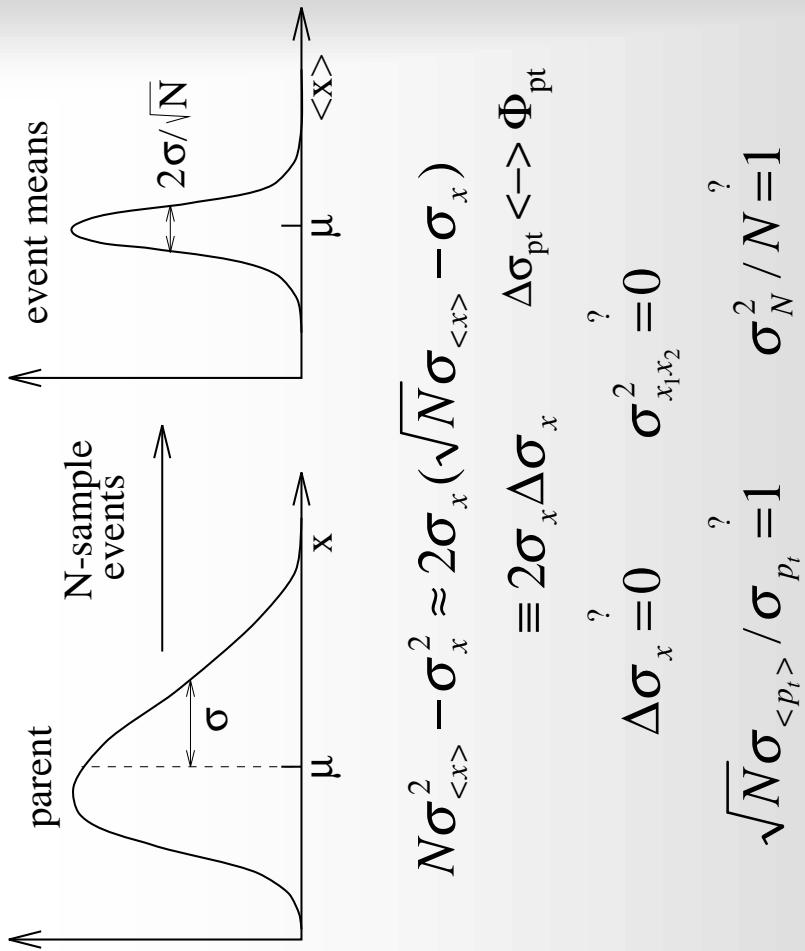
- A phase boundary separates regions with different densities of DoF
 - At the phase boundary long-range correlations may further reduce the number of DoF: critical fluctuations
 - Following sudden traversal a system may ‘remember’ two aspects of the phase boundary: DoF difference and/or critical correlations
- A larger number of DoF may persist through fluctuation suppression# and/or *incipient lattice* formation#
 - The correlation structure of heavy-ion collisions may reveal essential details of the QCD phase boundary

* S. Jeon and V. Koch, Phys. Rev. Lett. **85** (2000) 2076-2079
M. Asakawa, U. Heinz, B. Mueller, Phys. Rev. Lett. **85** (2000) 2072-2075

T.A. Trainor, hep-ph/0005176

The Central Limit Theorem

- The elementary central limit theorem (CLT) relates a sample-mean distribution variance to the variance of the parent distribution given two assumptions: 1) independent samples and 2) fixed parent
- The CLT provides an essential reference for variance comparisons *via difference factors* $\Delta\sigma_x$
- Deviations from the CLT are caused by pair correlations and/or dynamical fluctuations in measures (*e.g.*, p_t) and particle-production processes
- Difference-factor tests can also be reformulated as *ratio comparisons with unity* for specific cases
- The CLT is more generally a statement concerning the scale invariance of a total-covariance matrix



Scale-dependent Covariance Matrices

- In a more general form the CLT implies that equilibrated dynamical systems can be described by a *total covariance* matrix $K_X(\delta_X)$ that is scale invariant over some interval $[\delta_{X_1}, \delta_{X_2}]$. Any CLT test is based on a difference of covariance matrices evaluated at scale endpoints

$$\begin{aligned}
K_P(\delta x_2) - N K_P(\delta x_1) &= \begin{pmatrix} \sigma_\Phi^2 - N\sigma_p^2 + 2\bar{P}\sigma_{\Phi>N}^2 & \bar{P}\sigma_{\Phi>N}^2 + \hat{p}^2(\sigma_N^2 - N\sigma_n^2) \\ \bar{P}\sigma_{\Phi>N} + \hat{p}^2(\sigma_N^2 - N\sigma_n^2) & \hat{p}^2(\sigma_N^2 - N\sigma_n^2) \end{pmatrix} \\
&= \mathbf{A}(\delta x_1, \delta x_2; (p, n) \otimes (p, n)) \\
&= \begin{pmatrix} A_{pp} & \bar{P}A_{pn} \\ \bar{P}A_{pn} & \hat{p}^2 A_{nn} \end{pmatrix} \quad \text{T.A. Trainor, hep-ph/0001148}
\end{aligned}$$

- Deviations from a CLT hypothesis correspond to net two-point correlations expressible as a matrix \mathbf{A} of covariance integrals
 - One can also study individual covariance integrals or their integrands in the form of *two-point density ratios* (a separate NA49 study)
 - The present NA49 global-variables analysis is based on comparisons of specific covariance matrix elements at particle and event scales

The NA49 Detector System

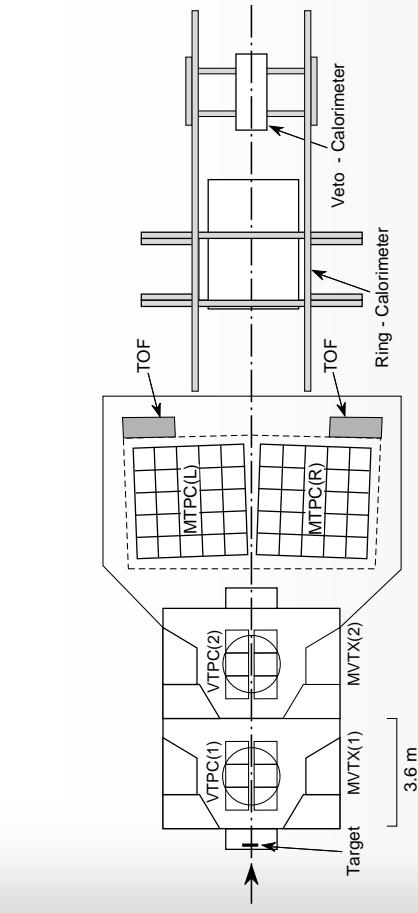
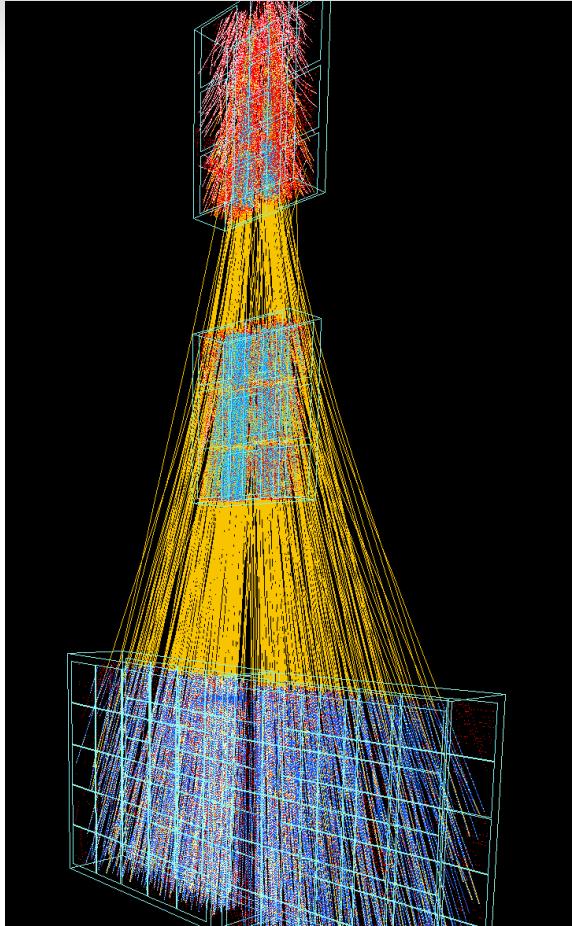


Fig. 1 : NA49 Experimental Layout

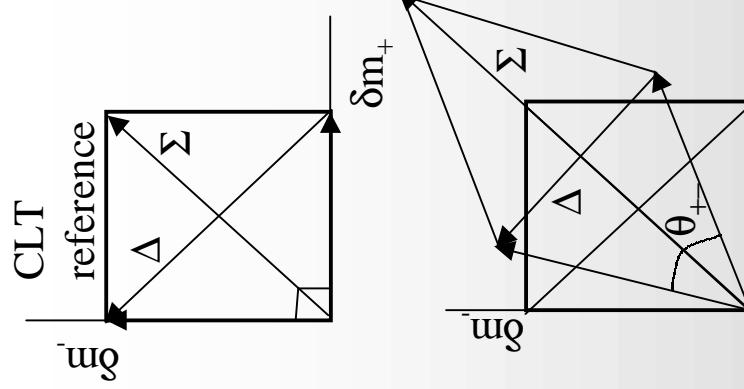
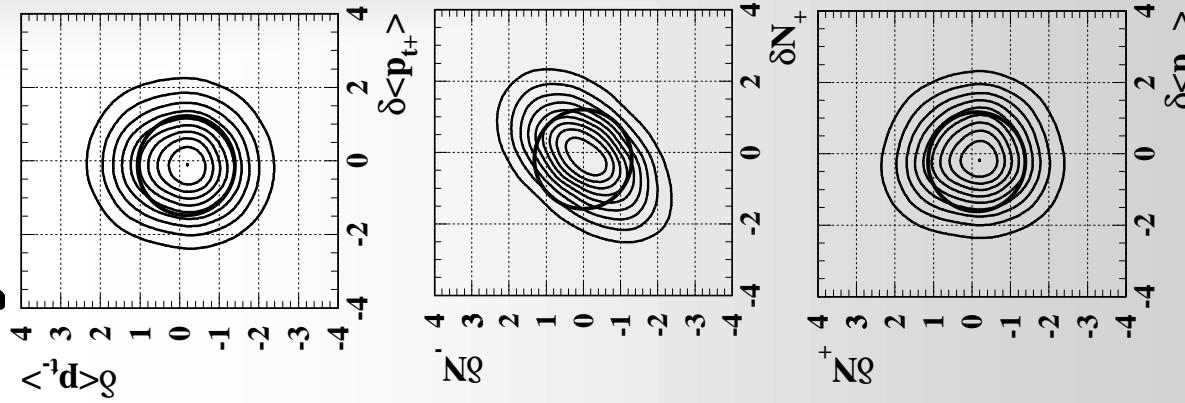


- Four time projection chambers (TPCs)
- Two vertex magnets with $B = 1.5 \text{ T}$
- Central events ($b < 3.5 \text{ fm}$) defined via veto calorimeter (100k for this study)
- Primary-vertex tracks defined via occupancy requirement in at least one vertex and one main TPC and dca cut to reduce weak-decay products
- Electron contamination eliminated by dE/dx cut
- Momentum acceptance defined by $0.005 < p_t < 1.5 \text{ GeV}/c$ and $4 < y < 5.5$
- Tracking efficiency $\sim 90\%$ before cuts
- Typical event statistics: $N_{ch} = 270$, $\sigma_N = 23$, $\sigma_{pt} = 0.28 \text{ GeV}/c$, $\bar{p}_t = 0.38 \text{ GeV}/c$

NA49 Global-variables Analysis

- Fluctuation vectors for each charge species and their sum ($\Sigma = \delta m_+ + \delta m_-$) and difference ($\Delta = \delta m_+ - \delta m_-$) are illustrated
- Vector fluctuation diagrams represent a central limit reference (upper) and *rms* fluctuation vectors with variance and covariance deviations (lower)
 - m_+ and m_- may be the same or different measures
- Data distributions for $\langle p_t \rangle$ fluctuations (top), N fluctuations (middle) and N- $\langle p_t \rangle$ correlations (bottom) are illustrated on the right
- Dark circles indicate nominal CLT variance references

J.G. Reid (NA49) analysis

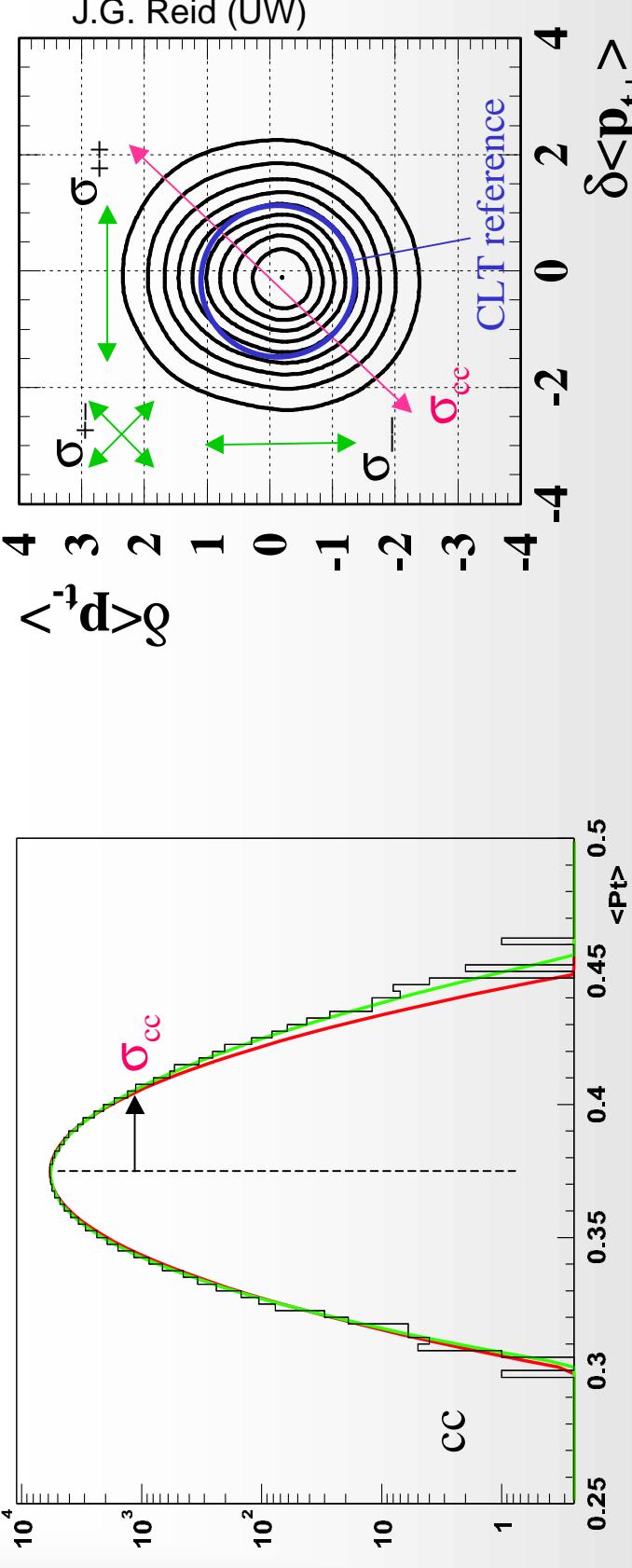


- Covariance:
- $$\sigma_{+-}^2 = \sigma_+ \sigma_- \cos \theta_+$$
- with
- $$\cos \theta_+ = r_+ = \text{linear correlation coefficient}$$
- variables are normalized:
- $$\delta m \rightarrow (m - \bar{m}) / \sigma_m (\text{CLT})$$

NA49 Global-variables Covariance Matrix

	P	N	<p>	P	p+	p-
var	100.6	579	0.000299	0.0807	0.09356	0.06118
sigma	10.0299551	24.062419	0.017292	0.28407745	0.305876	0.247346
mean	101.9	272.22	0.37433	0.37432959	0.41314	0.3276
calc1	100.58	100.7160669	579	0.000295	0.28529595	1.218497
calc2			0	0.28454773	0.470272	-0.299762
calc3				0.28431283	0.235379	0.298687
			P<p>	P+p-		
covar		213.35	-0.01262	0.0761		0
cosigma		14.6065054	-0.112339	0.275862		0
corcoff		0.884004295	-0.030331	0.438783		0
covarcalc		213.35	-0.01279	0.0748		
corcalc		0.884004295	-0.03074	0.431286		
			P-	N+	sqrtN+	N-
var	49.28	24.48	214		160	
sigma	7.01997151	4.9477268	14.62874	12.20246	12.6491106	0.000608
mean	61.52	40.4	148.9		11.10405	0.024658
calc	49.24798	49.3803744	24.51632	24.57235	123.3	0.413
						0.328
			P+N+	P+P-	P-N-	N+N-
covar	87.77	13.41	52.19	33.02	102.5	40.37
cosigma	9.36856446	3.661966685	7.2242647	5.746303	10.12423	6.35373906
corcoff	0.85468063	0.386089261	0.8339146	0.45621	0.553933	0.45463557
corcalc	0.85325989	0.386918321	0.83514	0.457174	0.45443432	0.012512
						0.510867
			ΣP	ΔP	ΣN	ΔN
var	1.684	0.7462	2.12388		0.61158	
sigma	1.29769026	0.8638287	1.457354		0.7820358	
mean	0	0	0		0	
calc				2.123903	0.6109514	
			$\Sigma P \Delta P$	$\Sigma N \Delta N$	$N \leftarrow p \rightarrow$	$N + \leftarrow p \rightarrow$
covar		0.03738		0.06994	-0.0133	-0.00509
cosigma		0.19333908		0.264462	0.115326	0.071344
corcoff		0.033345761		0.061367	-0.042642	-0.014111
corcalc					0.069779	

$\langle p_t \rangle$ Fluctuations - Distributions



- The 1D $\langle p_t \rangle$ distribution is compared visually to gaussian and gamma *inclusive reference* distributions
- The 2D $\langle p_t \rangle$ distribution is analyzed to extract charge-dependent (co)variances and related difference factors
- The σ_{cc} difference factor measures **dynamical fluctuations (DF)***
- Difference factors for other charge combinations probe **isospin dependence (IS)**

* H. Appelshaeuser, et al., Phys. Lett. **B459** (1999) 670

M. J. Tannenbaum, QM2001 poster session B: P107
T. A. Trainor

$\langle p_t \rangle$ Fluctuations - Algebra

From the basic CLT variance comparison we define
difference factors $\Delta\sigma_{pt}$ (or Φ_{pt}^*) for cc , $++$ and $--$

cc is undifferentiated
charge-charge

$$\overline{N}\sigma_{\langle p_t \rangle}^2 - \sigma_{p_t}^2 \approx 2\sigma_{p_t}(\sqrt{\overline{N}}\sigma_{\langle p_t \rangle} - \sigma_{p_t}) \equiv 2\sigma_{p_t}\Delta\sigma_{p_t}$$

For the covariance comparison $\Delta\sigma_{+-}$ we define

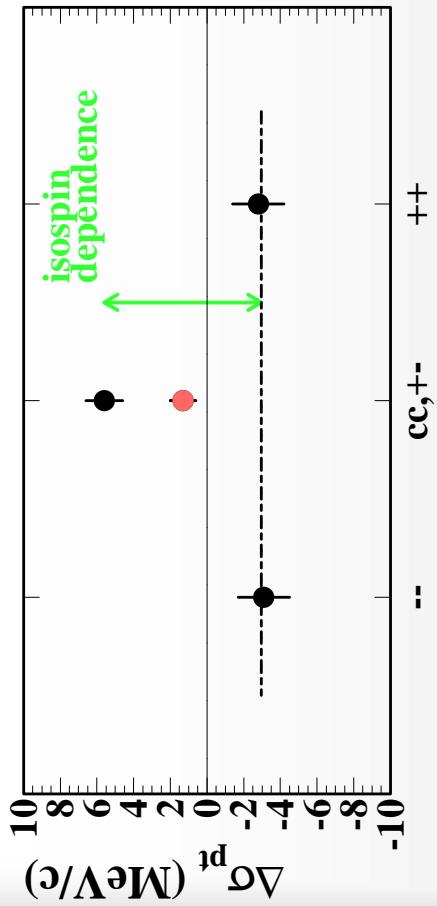
$$\Delta\sigma_{+-} \equiv \sqrt{N_+ N_-} \sigma_{\langle p_{t+} \rangle \langle p_{t-} \rangle}^2 / 2\sqrt{\sigma_{p_{t+}}\sigma_{p_{t-}}}$$

Difference factors for different charge combinations
should satisfy the following constraint

$$N\sigma_{p_t}\Delta\sigma_{cc} = N_+\sigma_{p_{t+}}\Delta\sigma_{++} + 2\sqrt{N_+ N_-} \sigma_{p_{t+}}\sigma_{p_{t-}}\Delta\sigma_{+-} \\ + N_-\sigma_{p_{t-}}\Delta\sigma_{--}$$

* M. Gazdzicki and St. Mrowczynski, Z. Phys. C54 (1992) 127-132

$\langle p_t \rangle$ Fluctuations - Analysis



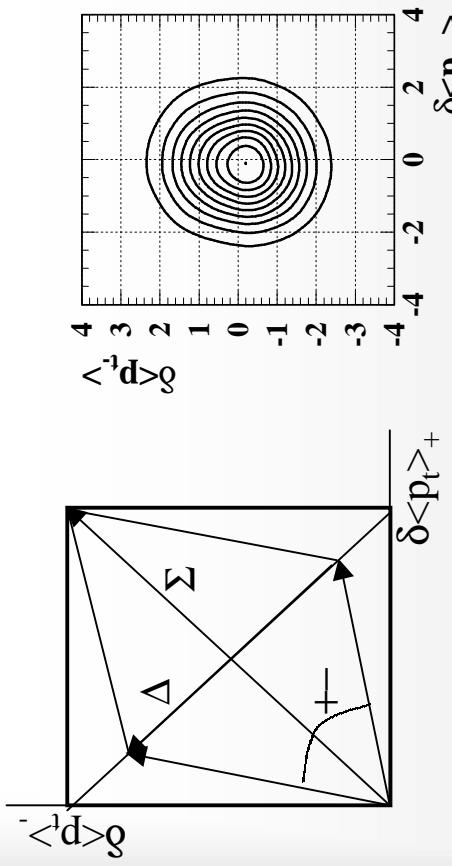
Compare these numbers
to $\sigma_{p_t} \sim 280$ MeV/c

- Difference factor **cc** measures **dynamical fluctuations**
- Other difference factors measure **isospin-dependent** effects
- Difference factors are corrected for short-range (anti)correlations (src) and two-track resolution (ttr)
- Difference factors are obtained by three different methods, which are equivalent if fluctuation data are ideally gaussian
- Small systematic variations (cases 1,2,3) correspond to nongaussian fluctuations (src/ttr-corrected values (2) - last table line - are adopted)

	J.G. Reid (UW)	cc
(MeV/c)	++	+-
σ_{p_t}	305.9	247.4
$\sqrt{N} \sigma_{\Delta p_t > 1}$	300.9	247.3
$\Delta \sigma_{p_t, 1}$	-5.0 ± 1.2	-0.1 ± 1.2
$\Delta \sigma_{p_t, 2}$	-6.1 ± 1.2	-1.6 ± 1.2
$\Delta \sigma_{p_t, 3}$	-7.2 ± 1.2	-1.4 ± 1.2
$\Delta \sigma_{p_t, 2, \text{corr}}$	-2.8 ± 1.2	-3.5 ± 1.2

NA49 ++/— charge asymmetry due to baryon-number asymmetry at SPS

$\langle p_t \rangle$ Fluctuations - Interpretation



- Relationship between fluctuations and correlations: reduced fluctuations imply *anticorrelation* while increased fluctuations imply correlation
- We observe that ++ and -- fluctuations are reduced while +- is increased, suggesting correlation of +- and anticorrelation of ++ and --
- This behavior is reminiscent of charge correlations in N-N *axial* phase space, a result of rapid emergence from a prehadronic state (string) without further hadronic equilibration
- $\langle p_t \rangle$ fluctuation results are represented qualitatively by the vector diagram
- ++ and -- variances are reduced relative to the CLT; +- covariance is increased (from zero) indicating **significant isospin (charge) dependence**
- $cc(\Sigma)$ agrees with the CLT expectation - **no dynamical fluctuations**

N_+, N_- Fluctuations - Algebra

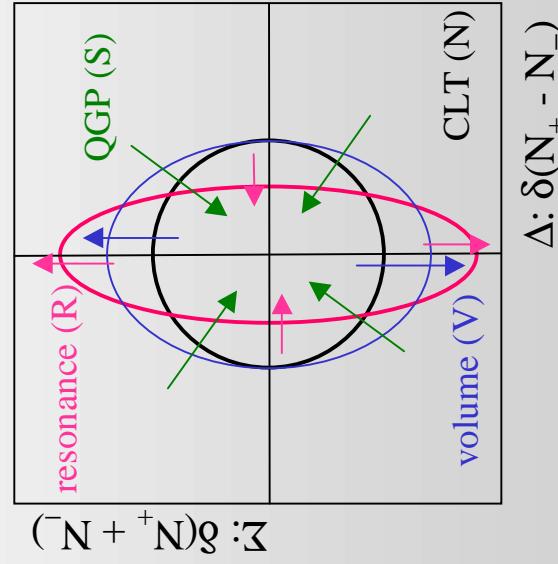
$$\sigma_{\Sigma}^2 = \sigma_+^2 + \sigma_-^2 + 2\sigma_{+-}^2$$

$$\sigma_{\Delta}^2 = \sigma_+^2 + \sigma_-^2 - 2\sigma_{+-}^2$$

$$\sigma_{\Sigma}^2 = N + \sigma_V^2 + \sigma_R^2 - \sigma_S^2$$

$$\sigma_{\Delta}^2 = N - \sigma_R^2 - \sigma_S^2$$

- Measured quantities are the variances and covariance for N_+ , N_-
- From these quantities sum Σ and difference Δ variances can be obtained
- Possible deviations from CLT reference N are shown in the diagram
- V** represents volume/trigger fluctuations
- R** represents hadron resonance correlations
- S** represents possible fluctuation suppression due to QGP formation



N_+, N_- Fluctuations - Predictions

- Volume fluctuations:
 - BH predict $\sigma_V^2 = 0.85N$, whereas DS predict $\sigma_V^2 = 0.40N$
 - Hadron resonances:
 - JK1 predict $\sigma_R^2 = 0.3N$ in σ_Δ^2 , whereas SRS predict $\sigma_R^2 = 0.5N$ in σ_Σ^2
 - Hadron resonances:
 - JK1 S. Jeon, V. Koch, nucl-th/9906074, Phys. Rev. Lett., **83** 5435 (1999)
 - SRS M. Stephanov, K. Rajagopal, E. Shuryak, hep-ph/9903292, Phys. Rev. **D60** 114028 (1999)
 - Fluctuation Suppression:
 - JK2 predict $\sigma_S^2 = 0.5N$, whereas AHM predict $\sigma_S^2 = 0.25N$
 - Available experimental information (two parameters) is not determining
 - Two possible approaches
 - assume σ_S^2 zero and compare σ_R^2 and σ_V^2 predictions to data or
 - accept predictions for σ_R^2 and σ_V^2 and establish constraints on σ_S^2 from data

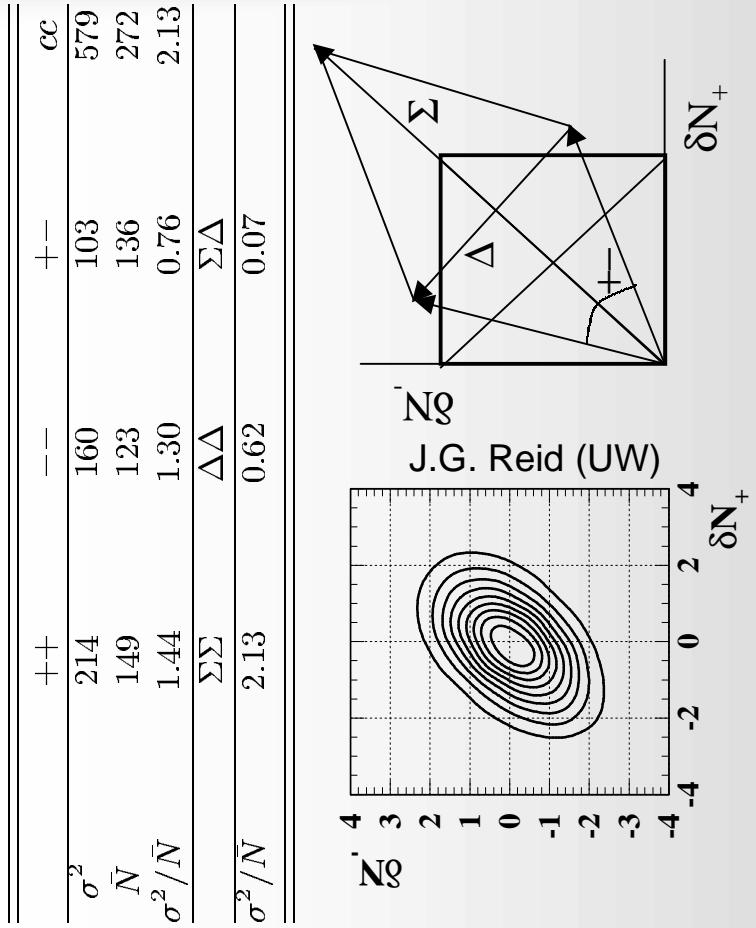
N_+, N_- Fluctuations - Results

- From $\sigma_\Sigma^2 = 2.13N$ and $\sigma_\Delta^2 = 0.62N$ we obtain constraints on $\sigma_V^2, \sigma_R^2, \sigma_S^2$
- Volume/trigger fluctuations(V) -

$$\sigma_V^2 = 0.75N + 2\sigma_S^2$$
 theory: $0.40N, 0.85N$
- Resonance correlations (R) -

$$\sigma_R^2 = 0.38N - \sigma_S^2$$
 theory: $0.3N, 0.5N$
- QGP suppression (S) -

$$\sigma_S^2 < 0.1N$$
 (given σ_R^2, σ_V^2 predictions)
- No significant evidence for QGP fluctuation suppression
- Finite covariance: $\sigma_{\Sigma\Delta}^2 = 0.07N$ suggests possible evidence for baryon-stopping fluctuations
- Lack of evidence for fluctuation suppression is consistent with combinatoric dilution of 3D rapid-traversal correlations projected to lower dimensionality (1D rapidity)*



*T.A. Trainor, hep-ph/0005176

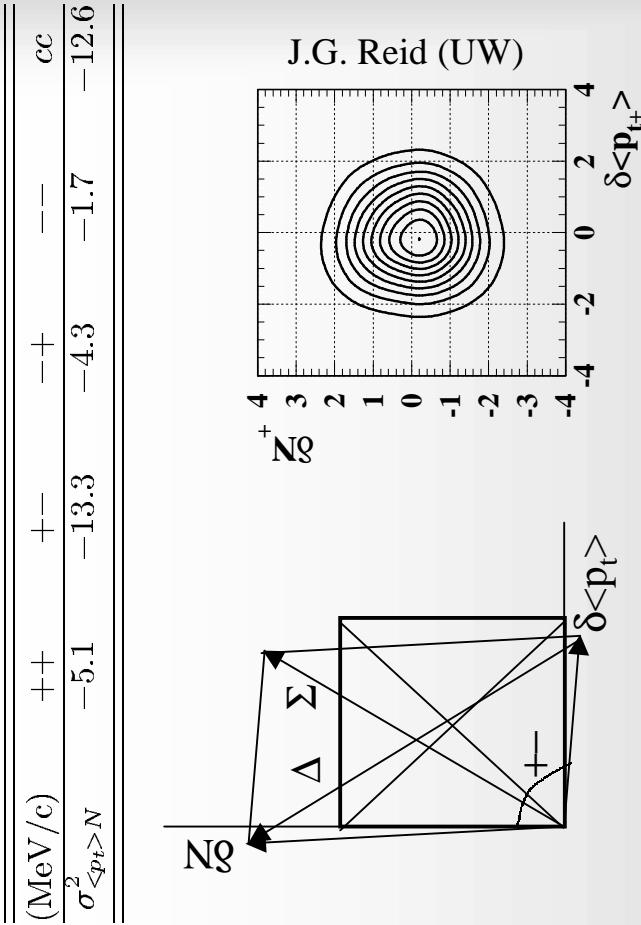
$\langle p_t \rangle$ -N Correlations

- Theory*: $\sigma^2_{\langle p_t \rangle N} = +18\text{-}36 \text{ MeV}/c$
- N-N collisions#: $\sigma^2_{\langle p_t \rangle N} = -75 \text{ MeV}/c$
- NA49: significant *negative* values for all four charge combinations: $++, --, +-,-+$ ($\sim 15\%$ of N-N)
- Consistency checked by constraint

$$\begin{aligned}\overline{N}\sigma^2_{\langle p_t \rangle N} &= \overline{N}_+(\sigma^2_{\langle p_{t+} \rangle N_+} + \sigma^2_{\langle p_{t+} \rangle N_-}) \\ &\quad + \overline{N}_-(\sigma^2_{\langle p_{t-} \rangle N_+} + \sigma^2_{\langle p_{t-} \rangle N_-})\end{aligned}$$

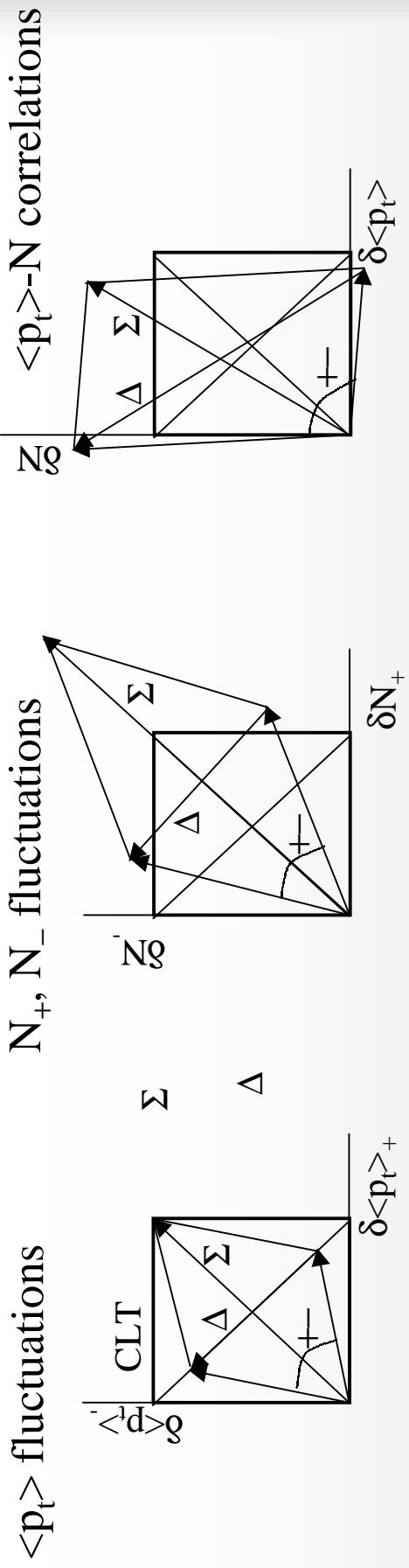
- Major disagreement with theory prediction
- Pb-Pb $\langle p_t \rangle$ -N result may be a relic of N-N superposition in HI collisions

- Momentum-multiplicity correlations could test hadronization universality:
Does momentum equilibration on particles depend on particle number?



* G. Baym, H. Heiselberg, Phys. Lett., **B469** 5435 (1999) 7
F. Liu, *et al.*, Eur. Phys. J. **C8** (1999) 649-654
T. Kafka, *et al.*, Phys. Rev. **D16**, (1977) 1261

NA49 Global-variables Summary



- **$\langle p_t \rangle$ fluctuations** - Difference factor for cc (Σ) is zero, indicating no dynamical fluctuations; factors for ++, -- and +- are nonzero, about 2% of inclusive σ_{pt} and indicate a pattern reminiscent of N-N axial phase space: spatial (anti)correlation of hadrons (pions) at chemical freezeout approximating a lattice
- **N_+, N_- fluctuations** - Strong deviations from the CLT are observed, consistent with predicted volume fluctuations and
- hadron resonance-gas charge correlations; experimental data are not determining, but there is no evidence for proposed fluctuation suppression associated with QGP formation; there is possible indication of baryon-stopping fluctuations
- **$\langle p_t \rangle$ -N correlations** - Observed small negative covariances for all charge combinations suggest residual momentum conservation in A-A collisions manifested more strongly in N-N collisions

Conclusions

- Fluctuation analysis was first directed to a search for QCD critical correlations as evidence for QGP formation, but *sudden traversal* of the QCD phase boundary can decrease fluctuations due to memory of a larger number of DoF as well as increase fluctuations due to critical correlations
- An initial NA49 global-variables $\langle p_t \rangle$ analysis has been extended to charge-dependent (isospin) effects, multiplicity fluctuations and $\langle p_t \rangle$ -N correlations
- The analysis reveals *significant isospin-dependent contributions* to $\langle p_t \rangle$ fluctuations
- Isospin-dependent fluctuations could be interpreted as signaling formation of a *lattice-like structure* at chemical freezeout due to rapid hadronization
- Multiplicity fluctuations deviate from a central-limit reference in agreement with hadronic resonance-abundance predictions
- Trigger/volume contributions to multiplicity fluctuations agree with several predictions, based however on inconsistent mechanisms
- Recent predictions of *fluctuation suppression* due to sudden traversal of the QCD phase boundary are *not* confirmed by observed multiplicity fluctuations
- Substantial $\langle p_t \rangle$ -N *anticorrelations* are observed, perhaps indicating residual momentum/energy-conservation effects noted previously in N-N collisions
- There is no indication of QGP exotica (bubbles, DCCCs) in NA49 fluctuations